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The supergravity dual of a theory with dynamical supersymmetry breaking

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Abstract: We study the large-N limit of a little string theory that reduces in the IR to $\mathrm{U}(N)$ $\mathcal{N}=1$ supersymmetric Yang-Mills with Chern-Simons coupling k. Witten has shown that this field theory preserves supersymmetry if $k \geq N/2$ and he conjectured that it breaks supersymmetry if $k < N/2$. We find a non-singular solution that describes the $k = N/2$ case, which is confining. We argue that increasing k corresponds to adding branes to this solution, in a way that preserves supersymmetry, while decreasing k corresponds to adding anti-branes, and therefore breaking supersymmetry.

Keywords: Brane Dynamics in Gauge Theories, AdS/CFT Correspondance.

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Contents

1. Introduction

In [1] Witten computed the index for $2+1$-dimensional SU(N) $\mathcal{N}=1$ super Yang-Mills in three spacetime dimensions. The theory also has Chern-Simons coupling k, which we take to be positive from now on. He found that the index is non zero for $k \geq N/2$ and zero otherwise. In the borderline case $k = N/2$ the index is precisely one, so that there is a unique ground state. This is a confining vacuum. It was further conjectured in [1] that in the case $|k| < N/2$ the theory spontaneously breaks supersymmetry.

In this paper we study a supergravity solution found by Chamseddine and Volkov [2]. After lifting this solution to ten dimension we interpret it as the supergravity dual of an NS 5-brane wrapped on S^3 with a twisting that preserves only $\mathcal{N}=1$ supersymmetry in 2+1 dimensions. At low energies this theory reduces to the three-dimensional theory considered in [1]. The Chern-Simons coupling is related to the flux of the NS 3-form H on S^3. The non-singular supergravity solution found in [2] corresponds to $k = N/2$. This gravity solution shows confinement. This is a solution where the S^3 that the NS-brane is wrapping becomes contractible in the full geometry, but the $\tilde{S}^3$ that is transverse to the branes is not contractible. The topology of the solution is consistent with the general picture advocated in [3, 4]. If $k = N/2 + n$ then we can add n-branes (or $|n|$ anti-branes if $n < 0$) wrapping the S^3 that is not contractible in the full geometry. If we wrap branes we preserve supersymmetry while we break supersymmetry if we wrap anti-branes.

Brane realizations of these $2+1$-dimensional Yang-Mills - Chern-Simons theories were studied in [5, 6], where also a description of supersymmetry breaking was given.

2. The field theory

Let us wrap IIB string theory NS 5-branes on S^3. In order to preserve supersymmetry we have to choose a twisting [7]. Since S^3 has three tangent directions the spin connection is in SU(2). Choosing a twisting amounts to choosing an embedding of this SU(2) into the R-symmetry group of the 5-brane theory. This group is SO(4) and it is the structure group of the normal bundle. We write $\mathrm{SO}(4) = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and we embed the spin connection into $\mathrm{SU}(2)_L$. It can be checked that this preserves $\mathcal{N} = 1$ supersymmetry in three dimensions. This twisting arises when we wrap a brane on an S^3 inside a G_2-manifold, such as the simplest G_2 non-compact spaces defined in [8]. If the radius of the sphere where we wrap the brane is very large, then at low energies (low compared to the six-dimensional gauge coupling constant) we can use the fact that we have a six-dimensional $\mathrm{U}(N)$ field theory on the NS 5-brane worldvolume. At energies low compared to the inverse radius of S^3 we get a $\mathrm{U}(N)$ theory in three dimensions. The three-dimensional theory contains only the gauge bosons and the gauginos. All other six-dimensional fields are massive. By adding a flux of the NS H field on the worldvolume S^3 we can induce a Chern-Simons coupling in three dimensions. This is easiest to see in the S-dual description where we know that on the D5-brane there is a coupling of the form [9]

$$\begin{aligned}\frac{1}{16\pi^3}\int_{\Sigma_6} B^{RR}\,\mathrm{Tr}[F\wedge F] &= -\frac{1}{16\pi^3}\int_{\Sigma_6} H_3^{RR}\,\mathrm{Tr}\left[A\wedge dA + \frac{2}{3}A^3\right]\\ &= -\frac{k_6}{4\pi}\int_{\Sigma_3}\mathrm{Tr}\left[A\wedge dA + \frac{2}{3}A^3\right]\,, \end{aligned}\tag{2.1}$$

where k_6 denotes the parameter appearing in the six-dimensional lagrangian to distinguish it from the final k that we will have in final $\mathcal{N} = 1$ theory in three dimensions after we integrate out all the six-dimensional Kaluza-Klein modes. These two are not the same because there is a shift in k when we integrate massive fermions. The final result is the three-dimensional k is given by $k = k_6 - N/2$. k is the coefficient of the Chern-Simons term appearing in the three-dimensional bare lagrangian, before we integrate out the gluinos. There is an additional shift by $-N/2$ when we integrate out the gluinos [1]. One way to understand this shift is as follows. Consider first the $\mathcal{N} = 2$ three-dimensional theory which is obtained by setting also the $\mathrm{SU}(2)_R$ R-symmetry connection equal to the spin connection equal to the gauge connection.[1] This arises if we wrap the brane on an S^3 in a Calabi-Yau, for example. In this theory the k appearing in the three-dimensional lagrangian is precisely the k appearing in (2.1). One way to see this is to notice that $\mathcal{N} = 2$ supersymmetry in three dimensions is the same as $\mathcal{N} = 1$ in four dimensions. Kaluza-Klein modes will be in multiplets which are like four-dimensional massive chiral multiplets reduced to three di-

[1] A theory with this matter content, but only $\mathcal{N} = 1$ supersymmetry, arises on the domain walls that separate different vacua of $\mathcal{N} = 1$ super-Yang-Mills in 3+1 dimensions [10].

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mensions. These multiplets contain two fermions one with positive mass and one with negative mass, so that when we integrate them out we do not shift k. We can further go to the $\mathcal{N}=1$ theory by setting the $\mathrm{SU}(2)_R$ gauge field to zero, this is something we can do continuously since there is no topology in an SU(2) bundle over S^3. This will leave us with only one fermion in three dimensions. Integrating out this last fermion produces the shift by $-N/2$ described above, see [11].[2] We see that k_6 is integer, so that k is automatically integer of half integer depending on whether N is odd or even.

To be precise, we start with a $\mathrm{U}(N) = \mathrm{U}(1) \times \mathrm{SU}(N)/Z_N$ theory. There is no shift in the level of the U(1) theory when we integrate out fields since no fields are charged under the U(1). So in the effective three-dimensional theory the level of the U(1) is still k_6.

The Witten index for the $\mathrm{U}(N)$ theory compactified on T^2 [1] can be written as[3]

$$I = \frac{(N+n)!}{n!N!}\,, \tag{2.2}$$

where $n = k - N/2 \geq 0$. Here we have used that we have a $\mathrm{U}(1)_{k_6} \times \mathrm{U}(N)_k/Z_N$ theory.

The description of the physics in terms of a weakly coupled three-dimensional theory is correct when the volume of S^3 is much larger than $(\alpha')^{3/2}$ so that the six-dimensional field theory description of the NS 5-brane theory becomes applicable. The three-dimensional theory will be weakly coupled at the Kaluza-Klein scale if we further impose that the three-dimensional effective coupling $g^2_{3,YM} N = N\alpha'/V_{S^3}$ is much smaller than $1/V_{S^3}^{1/3}$. This comes from demanding the effective energy scale at which the 't Hooft coupling becomes large should be smaller than the inverse size of the S^3, which is the Kaluza-Klein scale.

As usual, validity of the supergravity solution will imply that the scale where the three-dimensional theory becomes strongly coupled and the typical masses of Kaluza-Klein excitations are of the same order of magnitude. Nevertheless, we can still use the index theory computation of [1] since the index does not depend on continuous parameters, and we can continue from weak to strong 't Hooft coupling by changing the volume of S^3.

The index computation in [1] shows that there is only one vacuum if $k = N/2$. This unique vacuum is also confining [1]. To be precise SU(N) was considered in [1], while here we have U(N). The difference is a free U(1). We expect to see the center of mass U(1) as a topological mode in the gravity solutions, analogous to the well known singletons of AdS.

[2]As a check that these arguments regarding the sign are right notice that we can also find a supergravity solution, by a trivial redefinition of the fields of the solution we will write, that describes, what we have called $k_6 = 0$, without modifying the field theory twisting. According to the previous argument this should correspond to $k = -N/2$ and we indeed find that the gravity solution is essentially the same.

[3]In an earlier version of the paper there was an error in this equation. We thank S. Gukov for pointing it out to us.

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It was further argued in [1] that for $k > N/2$ the IR physics will be described by a bosonic Chern-Simons theory. The formulas in [1] are consistent with the idea that this bosonic Chern-Simons theory is $\mathrm{SU}(N)_{k-N/2}$. In the case of $k \gg N$ this was argued by integrating out the fermions. In our case we will consider $|k - N/2| \ll N$ and we will see that the IR physics is consistent with this IR behavior up to a level-rank duality.

3. The gravity solution

Now we look for a gravity solution that describes the near horizon geometry of a system of branes wrapping S^3, in the spirit of [12, 13, 14]. In order to think about the gravity solution it is convenient to use the truncation of the ten-dimensional equations to seven dimensions considered in [15]. The seven-dimensional theory is the minimal gauged supergravity in seven dimensions with an SU(2) gauge group. This SU(2) gauge group correspond to the $\mathrm{SU}(2)_L \subset \mathrm{SO}(4)$ R-symmetry group of the NS 5-brane. This seven-dimensional theory contains a metric, a dilaton, the SU(2) gauge fields and a three form field strength h plus their superpartners.

We will consider a supergravity solution which obeys the following boundary conditions for large ρ

$$
\begin{aligned}
ds^2_{7,\mathrm{str}} &\sim dx^2_{2+1} + \alpha' N \left[d\rho^2 + R^2(\rho) d\Omega_3^2\right], \\
\frac{1}{(2\pi)^2}\int_{S^3_\infty} h &= k\,, \\
A^a &\sim \frac{1}{2} w^a_L\,, \\
\phi &\sim -\rho\,,
\end{aligned}
\tag{3.1}
$$

where we have written the seven-dimensional metric in string frame. w^a_L are the left invariant one forms of S^3. The ρ dependence of the radius of S^3 can be interpreted as arising from the renormalization procedure in the "little string theory". We will see that the dependence on ρ will be linear. This effect was already observed in [13] and also in [16] for NS 5-branes wrapping S^2, where it was interpreted as the running of the coupling. In this case we can say something similar. If we think of e^ρ as the scale, then the dependence of the volume is "logarithmic". This statement should be taken with care since defining the UV/IR correspondence for NS 5-branes is tricky [17].

In [13] some solutions to the seven-dimensional equations were considered, precisely with this problem in mind. Those authors took the gauge fields to be independent of ρ and found that the spacetime develops a "bad" (according to the criterion of [18]) singularity at the origin. In a recent paper, Chamseddine and Volkov [2] found a non-singular solution for these equations for the case $k = N/2$. We will justify this value for k later in the section. The solution in [2] was written as a solution of $\mathcal{N} = 4$ SU(2) $\times$ U(1) gauged 5-d supergravity [19], but it can be shown using the formulas in [20, 21] that this solution lifts up to a solution of seven-dimensional gauged supergravity, which in turn can be lifted to ten dimensions, using [22, 23, 24].

The solution in [2] can be written as

$$\begin{aligned} ds^2_{\rm str,7} &= dx^2_{2+1} + \alpha' N \left[d\rho^2 + R(\rho)^2 d\Omega_3^2 \right] , \\ A^a &= \frac{w(\rho)+1}{2} w^a_L , \\ h &= N \left(w^3(\rho) - 3w(\rho) + 2 \right) \frac{1}{16} \frac{1}{6} \epsilon_{abc} w^a w^b w^c , \end{aligned} \tag{3.2}$$

where w^a are left invariant forms on S^3 and the dilaton is a function of ρ.

In [2] the system of equations coming from solving the BPS conditions was reduced to solving a single equation. It was then shown that there exists a solution to that equation with the prescribed boundary conditions. The solution is such that $R(\rho)$ decreases uniformly as ρ decreases and it becomes zero at $\rho = 0$. The full details of the solution can be found in [2] and in appendix A. The behavior of the fields at infinity is $R^2(\rho) \sim 2\rho$, $w(\rho) \sim 1/4\rho$, $\phi = -\rho + \frac{3}{8} \log \rho$. The behavior of R and the dilaton matches precisely with that of [13], but w was set to zero in [13]. We can interpret this dependence of w as function of ρ also as arising from a "renormalization" procedure in the little string theory. In other words, this behavior of w is independent of the boundary conditions related to the IR physics and arises purely from the UV region of the solution, i.e. we can find it by expanding the equations given in appendix A for large ρ. It arises due to the fact that there is an H flux over the worldvolume and when the S^3 is not infinite, then we need to modify slightly the spin connection to maintain supersymmetry. The behavior of the solution at the origin is $\phi = \phi_0 + o(\rho^2)$, $R^2 = \rho^2 + o(\rho^4)$ and $w = 1 + o(\rho^2)$. This implies that the gauge fields are trivial at the origin, up to a global gauge transformation and that the metric near $\rho = 0$ looks like the metric of R^4 so that this is a completely non-singular solution. Since the solution is non singular and the warp factor and dilaton are bounded at $\rho = 0$ we conclude, using the general arguments in [25], that we have a mass gap and confinement. Note that quarks are D-strings coming from the boundary.

The solution has one continuous parameter, ϕ_0, which is the value of the dilaton at the origin. The dilaton decreases monotonically as ρ increases [2]. This value of the dilaton at the origin is related to the ratio of the tension of the confining string to the mass of the Kaluza-Klein excitations

$$T = \frac{1}{2\pi\alpha'} e^{-\phi_0} , \qquad M^2_{KK} \sim \frac{1}{N\alpha'} . \tag{3.3}$$

In order to decouple the three-dimensional theory from the six-dimensional little string theory we need to take the ratio T/M^2 to zero. This can be achieved by taking $\phi_0 \to \infty$. In this case we need to do an S-duality (at least close to the origin of the solution) in order to analyze the gravity geometry. After we do this we find that the validity of the supergravity solution implies that $T/M_{KK}^2 \gg 1$ so that we cannot take this limit. Of course, if we could solve strings on RR backgrounds we could take this limit.

It is important to understand how this solution manages to be non singular. We see that the S^3 contracts to nothing. In order for this to be non singular we first need to remove the gauge fields that provide the twisting as we go from $\rho = \infty$ to $\rho = 0$. This is not a problem since an SU(2) bundle on S^3 can be continuously deformed to a trivial bundle. But we also have a flux of h on the S^3. So we need to remove it for the S^3 to shrink smoothly. This can happen since the field h obeys a modified Bianchi identity in the seven-dimensional theory of the form[4]

$$dh = N\frac{1}{2}\,\mathrm{Tr}\,F \wedge F = N\frac{1}{4}F^a F^a \,. \tag{3.4}$$

As we move in the radial direction and we deform the gauge fields we change the flux of h. We can see that the integrated change is precisely $N/2$ if w in (3.2) increases monotonically from $w = 0$ at infinity to $w = 1$ at the origin, independently of the precise form of w.[5] We see that only in the case that $k = N/2$ can we cancel completely the flux at $\rho = 0$ and obtain a non-singular solution. If $k \neq N/2$ we need to introduce extra sources for h at the origin. These are two-branes from the seven-dimensional point of view. In ten dimensions they are NS 5-branes wrapped on the internal $\tilde{S}^3$, the $\tilde{S}^3$ transverse to the original branes. These extra sources are branes for $k > N/2$ and anti-branes for $k < N/2$. If $|k - N/2| \ll N$ we can neglect the backreaction of these extra branes on the geometry. In order to show that the branes preserve supersymmetry while the anti-branes break it we need to consider the action of the ten-dimensional Γ matrices on the supersymmetry spinor. The supersymmetries preserved by an NS 5-brane in flat space obey

$$\Gamma^{1234}\epsilon_L = \epsilon_L\,, \qquad \Gamma^{1234}\epsilon_R = -\epsilon_R\,, \tag{3.5}$$

where 1234 are the indices labeling the four directions transverse to the brane. L and R label the two ten-dimensional spinors arising from the left and right movers on the superstring. The sign in (3.5) is reversed in both terms if we consider an anti-brane. In our background, (3.2), all ϵ_Rs are broken. Using the explicit form of the spinors given in [2] one can check that the supersymmetry generating spinor at the origin is such that locally on the brane the preserved spinor obeys the (3.5). This implies that only one sign of the brane charge preserves supersymmetry.

[4] In [15] this came from a Chern-Simons coupling involving the three form potential whose fieldstrength is dual to h. We will later derive this formula directly.

[5] The fact that the gauge fields performing the twisting are related to half instantons, or merons, was also noticed in [13].

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Now let us understand more precisely why the solution in [2] (after the uplifting) corresponds to $k = N/2$. For this it is necessary to study some aspects of the ten-dimensional solution. The dilaton in (3.2) is the same as the ten-dimensional dilaton, the rest of the ten-dimensional solution can be written, using [22, 23, 24], as

$$
\begin{aligned}
ds^2_{\rm str,10} &= ds^2_{\rm str,7} + \alpha' N \frac{1}{4}(\tilde{w}^a - A^a)^2 \,, \\
H &= N \left[-\frac{1}{4}\frac{1}{6}\epsilon_{abc}(\tilde{w}^a - A^a)(\tilde{w}^b - A^b)(\tilde{w}^c - A^c) + \frac{1}{4}F^a(\tilde{w}^a - A^a) \right] + h \,,
\end{aligned} \tag{3.6}
$$

where A^a and h are the seven-dimensional gauge fields and h field respectively. Their form on our solution is that indicated in (3.2). $\tilde{w}^a$ are the left invariant one forms on $\tilde{S}^3$, which is the three sphere transverse to the branes. Note that from (3.6) and $dH = 0$ we can derive (3.4). The topology (but not the metric) of the seven-dimensional space spanned by $\rho, w^a, \tilde{w}^a$ is asymptotically that of a cone whose base is $S^3 \times \tilde{S}^3$. When we talk about a three sphere we have to be very clear about which three sphere we are talking about. If we think about the three spheres as SU(2) group elements g and $\tilde{g}$, then we can consider spheres parametrized by $\hat{g}$ which is embedded in this space as

$$
g = \hat{g}^n \,, \qquad \tilde{g} = \hat{g}^m \tag{3.7}
$$

with n and m two arbitrary integers. The sphere transverse to the N 5-branes whose worldvolume theory we have been talking about is the sphere given by (3.7) with $n = 0$, $m = -1$ (the minus sign comes because we want the flux to be N and not $-N$). The sphere that the branes are wrapping is (3.7) with $n = 1$, $m = 0$. We find that the flux of H given in (3.6) with h as in (3.2) is N. We conclude from this that $k_6 = N$ so that $k = N/2$ for this solution. We have been careful to specify this sphere because by doing a global gauge transformation with winding number ν on the SU(2) gauge fields that implement the twisting we can change what we call the flux of H on the worldvolume by $N\nu$. This happens because a gauge transformation corresponds to the coordinate transformation on $\tilde{g} \to \tilde{g} g^\nu$. This gauge transformation will also change the net difference between the number of positive and negative mass fermions on the brane worldvolume.[6] This implies that the shift in the level for the SU(N) Chern-Simons term that occurs when we integrate out the massive 6-d Kaluza-Klein states depends on the gauge we choose for the connection on the normal bundle in such a way that the final three-dimensional k is independent of the gauge. As another check that we have the correct identification of k we note that we can trivially obtain another solution

[6] Under a gauge transformation of the connection on the normal bundle the net number of eigenvalues of the Dirac operator on S^3 that change sign is ν, the winding number of the gauge transformation. This is argued as in [1, section 2.1]. The only difference with the calculation performed in [1], is that our fermions are in the fundamental representation of SU(2).

from the solution in [2] by changing $h \to -h$ and $w \to -w$ in the present solution keeping everything else fixed. The second solution has essentially the same geometry. Calculating the fluxes we now obtain $k_6 = 0$ and $k = -N/2$ after we integrate out the Kaluza-Klein fermions on S^3. The fact that we also obtain a confining solution fits nicely with the fact that physics should be symmetric under $k \to -k$.

When we have extra branes in the geometry in the IR, i.e. , if $n \equiv k - N/2 \neq 0$ then the theory on these n-branes determines the IR dynamics of the original theory. The value of H on the worldvolume of the branes is essentially of the same order as the curvature of the sphere and the explicit form for the 5-brane action is not known in this case, see however [26, 27]. Nevertheless we can say that the action will describe a $\mathrm{U}(n)$ gauge theory with level $N + o(n)$. The mass coming from the Chern-Simons term is of the same order of magnitude as the mass of the Kaluza-Klein states. Furthermore, the three-dimensional gauge coupling constant at the Kaluza-Klein scale goes as n/N and it is therefore weak in the limit we are considering. Let us consider first the case $n > 0$. The theory on the branes is then supersymmetric. At low energies we will have a purely bosonic Chern-Simons theory (if $n > 0$). The precise formula for the index (2.2) suggests that this bosonic theory is $\mathrm{U}(1)_{N+n} \times \mathrm{SU}(n)_N/Z_n$ Chern-Simons which agrees with what we get up to shifts of order n in the level. To verify these shifts in the level we would need to do a more careful analysis of the 5-brane action. It is interesting that this theory is the level rank dual [28] to the naive guess for the low energy dynamics based on integrating out the fermions in the original boundary field theory which is $\mathrm{U}(1)_{N+n} \times \mathrm{SU}(N)_n/Z_N$. If $n \neq 0$, a D1-brane coming from the boundary can end on the n NS-5 branes. This implies that Wilson loops can have non-zero expectation values for large areas. Furthermore, for large areas the expectation values of such Wilson loops is given by this bosonic Chern-Simons theory. Note that we can put the theory on T^2 by compactifying the spatial directions. This does not introduce any singularity in the gravity solution. The index in the gravity picture is then given by the number of states in the low energy Chern-Simons theory.

In the $n < 0$ case we cannot say precisely what happens with the IR dynamics on the branes since we also have massless fermions. So at low energy we have massless fermions in the adjoint of $\mathrm{U}(n)$ interacting via Chern-Simons interactions. The $\mathrm{U}(1)$ part of the fermions is exactly massless and that is the goldstino.

It is also interesting to consider the magnetic 't Hooft operator $\mathcal{O}(P, w)$ described in [1], which is defined by removing a point P from spacetime and inserting a nontrivial magnetic flux w on the sphere surrounding P. In four dimensions this object is the 't Hooft *loop* operator. It was observed in [1] that this operator only exists if $k - N/2 = 0$ modulo N. This operator corresponds, essentially, to a D3-brane coming from the boundary and wrapping the worldvolume three sphere, S^3. More precisely to the sphere given by (3.7) for $n = 1$ and m such that the flux of H over the sphere vanishes. We need that the flux of H vanishes on any D-brane as was explained in [29]. It is possible to choose such a sphere only if $k - N/2 = 0 \mod N$. We see that the expectation value of $\langle \mathcal{O}(P, w) \rangle$ is non zero if $k = N/2$ since we can wrap the D3 on S^3 times the radial direction. More precisely, we have to wrap the brane on the S^3 given by (3.7) with $n = m = 1$ which is the topologically contractible sphere in the geometry that we are considering.

Notice that supersymmetry breaking, which was a dynamical, non-perturbative effect from the field theory point of view, becomes spontaneous symmetry breaking, visible in the fact that we cannot find a supersymmetric classical solution. The fact that a non-perturbative effect on one side of a duality is the same as a classical effect on the other is familiar. This is true both if we view these solutions as classical solutions in string theory where the expansion parameter is $g_s = e^{\phi(\rho=0)}$, since 5-branes are classical solutions, or if we view N as the expansion parameter and we interpret n/N as a small parameter appearing in the lagrangian (In gravity it is a parameter appearing in the boundary conditions for the fields in the gravity solution).

We can compute the vacuum energy in this solution. A naive attempt at computing the vacuum energy would involve computing the classical supergravity action on this configuration. This action is divergent if we just use the most naive cutoff procedure which is to integrate the action up to $\rho = \rho_c$. This fact is very familiar and in AdS examples one can remove the divergent terms by local counterterms [30]. In our solution the divergent terms go like $e^{2\rho_c}$ times an infinite series in $1/\rho_c$. These divergent pieces depend on k, through the asymptotic value of H on the worldvolume S^3. It is clear that this brute force procedure for extracting the finite piece will not work, since it involves computing an infinite number of terms. If we interpret e^{ρ_c} as a scale, then powers of $1/\rho_c$ look like the familiar logs that we expect in an asymptotically free theory. Of course this problem appeared because we considered a non-supersymmetric cutoff procedure. A supersymmetric cutoff procedure would automatically give zero for a supersymmetric solution. The value of the classical action on a solution gives, after integrating by parts and using the dilaton equation of motion,

$$S \sim \int \sqrt{g} e^{-\phi} \partial_\rho e^{-\phi} |_{\rho=\rho_c} \,, \tag{3.8}$$

where we integrate over a nine-dimensional surface at $\rho = \rho_c$. The counterterms that we are allowed to include to render this expression finite are arbitrary functions of the boundary values of the fields, in our case the fields are the ones appearing in the ansatz (3.2) ϕ, w, R. If the solution is supersymmetric we can express the derivative appearing in (3.8) in terms of the values of the field at that point. So it is clear that we can subtract the whole expression to render the vacuum energy finite. This argument shows that the vacuum energy for supersymmetric solutions is zero, including the solution with n-branes when $n > 0$. We now consider the solution with $|n|$ anti-branes, $n < 0$, then we compute the vacuum energy as follows. Consider first $n = -1$. We could formally obtain a supersymmetric solution with the right boundary conditions if we put a brane with negative tension and negative charge at the origin. The vacuum energy of such a solution would be zero by the previous argument. This differs from the solution we had by the addition of a pair of branes with positive tension, one with positive charge and one with negative charge. So we conclude that the vacuum energy is given as *twice* the energy of a brane wrapping S^3. In other words the vacuum energy for $n < 0$ is

$$T_3 = 2|n| \frac{1}{(2\pi)^5 \alpha'^3} e^{-2\phi_0} V_{\tilde{S}^3} = \frac{|n| e^{-2\phi_0} N^{3/2}}{(2\pi)^3 \alpha'^{3/2}} \,. \tag{3.9}$$

This calculation is valid only if $|n| \ll N$ where we can neglect the interactions of these branes. If $n \sim N$ we would have to take into account the backreaction of the branes and the problem appears to be more complicated since it involves finding a non-supersymmetric gravity solution. As we remarked above, results like (3.9) are not expected to reflect the true vacuum energy of the decoupled three-dimensional theory since this gravity solution is not valid in that case.

Notice that these solutions display the general features described in [3, 4] regarding topology change due to the branes. Before we put the NS-5 branes we start with a G_2-manifold which is asymptotically a cone with a base $S^3 \times \tilde{S}^3$. The space is such that at the origin S^3 has finite volume while $\tilde{S}^3$ is contractible. In the final solution, after we take into account the backreaction of the branes, the fate of the two S^3s has been interchanged. $\tilde{S}^3$ is not contractible but S^3 is contractible. So the final topology (but not the metric) is that of a G_2-manifold on the other side of the "flop" transition described in [31]. It would be nice to see if there is a description in terms of an effective superpotential, of the type described in [4], for these systems with $\mathcal{N} = 1$ supersymmetry in $2 + 1$ dimensions.

Another system that is interesting to study, which will probably produce results similar to these, is 5-branes wrapped on S^3 with the twisting that preserves $\mathcal{N} = 2$ supersymmetry in $2 + 1$ dimensions.

In [32] configurations of D2-branes and NS-5 branes wrapped on S^3 were considered and solutions were found that have similar features to the solutions described here. It is possible that one can repeat the analysis of this paper for those solutions.

It will also be interesting to find gravity examples of supersymmetry breaking for theories holographically dual to 4-d gauge theories.

While this paper was in preparation we received [33] which discusses the uplifting of the solution in [2] to ten dimensions.

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A. The CV solution and embedding in 10d

In this appendix we present a few details of the solution found by Chamseddine and Volkov [2] and its uplifting.[7] The bosonic fields of $\mathcal{N} = 4$ SU(2) $\times$ U(1) gauged 5d supergravity are the metric, an SU(2) gauge field A^a, an U(1) gauge field a, with curl $f = da$, a pair of 2-form fields and the dilaton. The two 2-form fields can be put to zero on-shell, and the U(1) gauge coupling can be set to zero too.

With the ansatz

$$\begin{aligned} ds^2_{\text{str},5} &= -dt^2 + \alpha' N \left[d\rho^2 + R(\rho)^2 d\Omega_3^2 \right] , \\ A^a &= \frac{w(\rho)+1}{2} w_L^a , \\ f &= Q(\rho) dt d\rho \end{aligned} \tag{A.1}$$

one can first solve for Q and obtain a set of equations for the general (non supersymmetric) case. [2] the looked at solutions to the BPS conditions, satisfying $L_a \epsilon = 0$ and $(\sigma_a + \tau_a)\epsilon = 0$, where L_a is the left-angular momentum operator on S_3, and σ_a and τ_a come in the definition of gamma matrices. The spacetime gamma matrices are

$$\gamma^0 = i\underline{\sigma}^3 \otimes \mathbf{1} , \qquad \gamma^r = \underline{\sigma}^1 \otimes \mathbf{1} , \qquad \gamma^a = \underline{\sigma}^2 \otimes \sigma_a \tag{A.2}$$

and the internal gamma matrices are

$$\Gamma_a = \underline{\tau}_2 \otimes \tau_a , \qquad \Gamma_8 = \underline{\tau}_1 \otimes \mathbf{1} , \qquad \Gamma_9 = \underline{\tau}_3 \otimes \mathbf{1} . \tag{A.3}$$

That means that the $(\sigma_a + \tau_a)\epsilon = 0$ condition will correspond in the uplifting to the twisting condition $(\omega_a + A_a)\epsilon = 0$, and the $L_a \epsilon = 0$ condition will correspond to the $\partial_\mu \epsilon = 0$ condition. Then ϵ is expressed in terms of 4 unknown functions, and from the condition that the BPS equations have a solution, one gets

$$\begin{aligned} \frac{dR}{d\rho} &= \frac{1}{3\sqrt{M}} \left[\frac{V^2}{64R^4} + \left(3(w^2-1)^2 - Vw\right) \frac{1}{2R^2} + w^2 + 2 \right] , \\ \frac{dw}{d\rho} &= \frac{4R}{3\sqrt{M}} \left[\frac{V}{32R^4}(1-w^2) + \frac{(2\kappa - w^3)}{R^2} - w \right] , \\ \frac{d\phi}{d\rho} &= \frac{3}{2} \frac{d\log R}{d\rho} - \frac{3}{2} \frac{\sqrt{M}}{R} , \end{aligned} \tag{A.4}$$

[7] We are using slightly different variables and normalizations than [2]. The relation between the variables is $\sqrt{8}R = e^\nu / r$, $\phi = -\frac{3}{2}\nu$, $dr/d\rho = \sqrt{8} e^{2\nu} \sqrt{M}$, $8\kappa = H$ where r, ν, H are defined in [2].

where

$$M = \left(\frac{V}{24R^2} - w\right)^2 + \frac{(w^2-1)^2}{4R^2} - \frac{2}{3}(w^2-1) + \frac{4R^2}{9}\,,$$
$$V = 2w^3 - 6w + 8\kappa \qquad \text{(A.5)}$$

and κ is an arbitrary constant that is related to the flux of h over S^3. More precisely it is $\kappa = k/N$. Note that eqs. (A.4)and (A.5) are symmetric under $\kappa \to -\kappa$ and $w \to -w$. By setting $\kappa = 1/2$, one gets a solution which is regular at $\rho = 0$. It can be expressed in terms of the variable $Y(w) = 8R^2 + w^3 + 4w - 2 - \frac{4}{w}$ which obeys

$$w^2 Y \frac{dY}{dw} = 4(w-1)^2 Y + 16(w-1)(2w+1)(w+2) \qquad \text{(A.6)}$$

and has a solution with asymptotics

$$Y = 8 + 28w + 92w^2 + \cdots\,, \qquad \text{as} \qquad w \longrightarrow 0\,,$$
$$Y = 12(1-w) + 4(1-w)^2 + 2(1-w)^3 + \cdots\,, \qquad \text{as} \qquad w \longrightarrow 1\,. \qquad \text{(A.7)}$$

The solution for the supersymmetry parameter ϵ is such that at $\rho = 0$ it obeys $\underline{\sigma}_3\epsilon = \epsilon$ and at $\rho = \infty$ it obeys $\underline{\sigma}_1\underline{\tau}_2\epsilon = \epsilon$.

With the gamma matrix embedding into 10 dimensions

$$\Gamma_A^{(10)} = \gamma_A^{(5)} \otimes \mathbf{1} \otimes \lambda_1\,,$$
$$\Gamma_i^{(10)} = \mathbf{1} \otimes \Gamma_i^{(5)} \otimes \lambda_2\,,$$
$$\Gamma_{11} = -\mathbf{1} \otimes \mathbf{1} \otimes \lambda_3\,. \qquad \text{(A.8)}$$

we find that $\Gamma^r\Gamma^{123} = -\underline{\sigma}^3$ and $\Gamma^r\Gamma^{1'2'3'} = -\underline{\sigma}^1\underline{\tau}^2\lambda_3$ and so we find that the brane wrapped on 1'2'3' preserves supersymmetry, as in the discussion following (3.5).

As for the embedding of the 5d fields into 7d, it is almost trivial, the string frame metric gets an extra term of the form $dx_8^2 + dx_9^2$, the gauge field stays the same, and the 4-form field strength in 7d becomes $F_{(4)} = f dx_8 dx_9$. In the formulation of a 3-form h used in the text, $h = e^{2\phi} *_{7,\mathrm{str}} F_{(4)}$. So the 7d solution is the one in (3.2), and then the uplifting to 10 d is given by (3.6).

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